An Exploration of Fact Fluency Development in 150,000 Elementary Students

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1. Perspectives or Theoretical Framework for the Research

Fluency with basic addition and subtraction facts is a critical skill that predicts later student achievement in mathematics (Baroody, Eiland, Purpura, & Reid, 2013; Codding & Martin, 2016; Geary, 2010; Geary et al., 2009; Gersten et al., 2009). Fact fluency is generally acknowledged as requiring both *accuracy* and *speed* when solving facts (Van der Ven, Segers, Takashima, & Verhoeven, 2017). Students must not only be able to find correct answers to facts, but to do so in an efficient manner. Being able to solve basic facts quickly is presumed to free working memory for other more demanding mathematical tasks, thus explaining the connection between facility with basic facts and later mathematics achievement.

Purpura, Baroody, Eiland, & Reid (2016) described fact fluency as progressing along a continuum. The highest level of fluency was indicated by students who provided an accurate response to an addition or subtraction problem in under 3 seconds, with no evidence of counting or explicit strategy use (see also Baroody et al., 2013). Lower levels of fluency were similarly described in terms of accuracy, observed strategy use, and speed. Purpura et al. (2016) subdivided speed levels into under 3 seconds (broken further into fluent speed without strategy use; nearly fluent speed with strategy use), 3-6 seconds, 6-15 seconds, or 15+ seconds with no response.

This continuum suggests a relationship between strategy use and the development of speed with basic facts, wherein the use of automatic recall is associated with a quicker speed, while the use of counting or other calculation strategies is associated with slower speed. Van der ven et al. (2017) describe a similar relationship between accuracy, speed, and strategy use. They argue that fluency with basic facts increases when students do one of three things: (1) improve retrieval speed, (2) improve calculation speed, or (3) move from calculation to retrieval, as retrieval is inherently faster.

This description aligns with collective wisdom on the development of fact fluency (Baroody, Bajwa, & Eiland, 2009; Gersten et al., 2009; National Research Council, 2001), which suggests that students progress through a series of procedural stages along the path to fact fluency: counting, use of meaningful strategies, and recall/memorization. The use of meaningful strategies involves drawing on known facts to derive others (such as using knowledge of 3 + 3 to solve 3 + 4), with the idea that some facts--such as doubles and addition of 1--serve as "helper facts" for learning others. Meaningful strategy use also includes using mathematical properties, such as the commutative property or the relationship between addition and subtraction, to solve categories of similar facts.

Current research examines how fact fluency can be improved through instructional programs that encourage meaningful strategy use (Baroody et al., 2013; Purpura et al., 2016). To better understand the development of fact fluency and instruction that supports it, this study explores fact fluency in over 150,000 elementary students. Using data on student accuracy and speed in a web-based fact practice game, we ask:

1. What are students' fluency levels with different groups of basic addition and subtraction facts?
2. In what order and combinations do students appear to become fluent with different fact groups?
3. Which fact groups, once mastered, appear to support fluency with other fact groups?

2. Data Sources or Evidence for the Research

The data for this study came from student use of the *Facts Workshop Game*, a web-based game available to users of *Everyday Mathematics,* an elementary mathematics curriculum published by McGraw-Hill Education. The game provides practice with addition, subtraction, multiplication, and division facts with a range of challenge levels, timed and untimed modes, and so forth. This study examines only data from versions of the game that focused on basic addition and subtraction facts in untimed modes.

The game uses a fact-family approach to practicing facts, in which students answer questions about related addition and subtraction facts at the same time. Students must answer a set number of questions in a level correctly in order to move on to the next level in the game. Within each level, there are three different fact groups presented: +/- 0 or +/- 1 facts (henceforth called 0s/1s facts), doubles facts

(such as 8 + 8 or 14 - 7), and facts ranging from 0 + 1 to 10 + 10 (henceforth called mixed facts). Students must correctly answer a set number of questions from each fact group within each level.

We received de-identified data on student use of the game from McGraw-Hill Education in fall of 2016. The data was structured around individual question attempts by individual students, with each attempt recorded as a separate row in the data table. Information recorded about each question attempt included the kind of fact (fact group), the time in milliseconds spent on the question, and whether the answer given was correct or incorrect. The data contained such information for 155,628 students.

3. Methods, Techniques, or Modes of Inquiry for the Research

Fluency Measurement. In previous work (Authors, 2017), we examined the overall level of student accuracy with different fact groups. For this study, we extended our probe to students' **fluency levels** (see Baroody et al., 2013 & Purpura et al., 2016), which we defined as the average amount of time, in seconds, that a student took to answer a question from a particular fact group *correctly*. Incorrect attempts were not included when examining fluency levels, as we define as fluency as including both accuracy and speed. Time to attain an incorrect answer therefore does not serve as a valid measure of fluency. The first question students answered in each gaming session was also excluded from the fluency level measure, as comparisons among the average time needed to answer the first four questions showed that students take significantly longer to answer the first question, presumably as they adapt to the new game level.

Drawing on Purpura et al. (2016), students were first sorted into four **fluency levels** for each fact group: (1) Fluent: Average time to provide correct answer is under 3 seconds, (2) Fast: Average time to provide correct answer is 3-6 seconds, (3) Medium: Average time to provide correct answer is 6-15 seconds, and (4) Slow: Average time to provide correct answer is more than 15 seconds. In examining the distribution of students across these groups, we found that only 6% of students achieved an average speed of 3 seconds or less on any fact group. Thus, in order to have a sufficient number of students in each group, we combined the two fastest groups into one group of students with average speeds of 0-6 seconds. We believe that, for this particular game, 3-second speeds may simply be too high a standard for many students to achieve, and thus we combined Fluent and Fast into one fluency level that we call "Fluent."

Method for Question 1. Our first question asked, "What are students' fluency levels with different groups of basic addition and subtraction facts?" To answer this, we examined differences in the percentages of students assigned to each fluency level across fact groups.

Method for Question 2. Our second question asked, "In what order and combinations do students appear to become fluent with different fact groups?" To answer this, we examined how many students were Fluent on each possible collection of fact groups (e.g., 0s/1s only, doubles only, 0s/1s and doubles, 0s/1 and mixed, and so on). Examining the groups with the greatest number of students provides insight into the order in which students acquire fluency.

Method for Question 3. Our third question asked, "Which fact groups, once mastered, appear to support fluency with other fact groups?" This question required looking at the relationship of fluency with 0s/1s and doubles to fluency on mixed facts. To address this, we conducted ANOVA tests and follow-up pairwise t-tests (Bonferroni adjustment) on the mean time to correctly answer mixed facts (a measure we refer to simply as *speed* on mixed facts) for students in various groups.

4. Results and/or Conclusions

Question 1: General Level of Student Fluency. Table 1 shows the distribution of students across fluency levels for each fact group. More students are Fluent with doubles than any other fact group, and more students are Fluent with 0s/1s than with mixed facts. For all three fact groups, the greatest number of students are in the Medium fluency level. For 0s/1s and doubles, the second-greatest number of students are Fluent, whereas for mixed facts, the second-greatest number of students are in the bottom fluency level (Slow). These results suggest that students are more fluent with doubles than 0s/1s, and more are fluent with 0s/1s than mixed facts.

Question 2: Order of Fluency Acquisition. Table 2 shows the distribution of students according to the fact groups on which they were Fluent. The greatest number of students were not Fluent with any fact group. The next two groups, ordered by number of students, are those Fluent on doubles only and on 0s/1s only, respectively. These groups are followed by students Fluent on both 0s/1s and doubles, then students Fluent on all facts. Less than 7% of students are Fluent on mixed facts without also being Fluent

Table 1. Distribution of Students across Fluency Levels by Fact Group.

Fact Group	Fluency Level	Number of students	Percent of students
0s/1s	Fluent	39,090	25.12%
	Medium	88,499	56.87%
	Slow	28,037	18.02%
Doubles	Fluent	49,873	32.05%
	Medium	85,543	54.97%
	Slow	20,210	12.99%
Mixed	Fluent	18,871	12.13%
	Medium	107,515	69.09%
	Slow	29,240	18.79%

on the other two fact groups. These results suggest that students generally first acquire fluency with either doubles or 0s/1s, with doubles coming first for more students. Then students acquire fluency with the other potential helper fact group (0s/1s or doubles) before acquiring fluency with mixed facts.

Question 3: Relationships between Fluency on Fact Groups. We examined the relationship between fluency with 0s/1s and doubles facts (the potential "helper facts" in meaningful strategy use) and speed on mixed facts (defined as average time to correctly answer mixed facts). Table 3 shows the average speed on mixed facts for students grouped by the helper fact groups on which they are Fluent. Students who are Fluent with both groups of potential helper facts are quickest to correctly answer mixed facts, followed by students Fluent with doubles only, followed by students Fluent with 0s/1s only, followed by students who

Table 2. Distribution of Students across Fluent Fact Groups.

Fluent Fact Groups	**Number of students**	**Percent of students**
None	86,230	55.41%
Doubles	21,945	14.10%
0s/1s	14,692	9.44%
0s/1s, Doubles	13,888	8.92%
0s/1s, Doubles, Mixed	8,477	5.45%
Doubles, Mixed	5,563	3.57%
Mixed	2,798	1.80%
0s/1s, Mixed	2,033	1.31%

are not Fluent with either group of helper facts. An ANOVA detected a significant difference across groups ($p < 0.001$, F-value: 16.37, df: 3), and follow-up pairwise t-tests showed significant difference between students who are Fluent with both 0s/1s and doubles and students who are not Fluent with either ($p < 0.001$) and between students who are Fluent with doubles and students who are not Fluent with either ($p < 0.001$). There were also significant differences between students who are Fluent with 0s/1s only and all three other groups ($p < 0.001$ in comparison to doubles and both 0s/1s and doubles; $p < 0.05$ in comparison to neither). There was no significant difference between students Fluent with both helpers and students Fluent with doubles.

In sum, being Fluent with doubles facts, regardless of whether it is accompanied by fluency with 0s/1s, appears to accompany quicker correct answers on mixed facts. Fluency with 0s/1s appears to accompany quicker correct answers on mixed facts more than having fluency with neither, but less than having fluency with doubles.

Table 3. Speed on Mixed Facts by Fluency with Helper Facts.

Fluent Fact Groups	**Mean Speed on Mixed Facts (s)**	**SD**
0s/1s, Doubles	8.02	40
Doubles	9.54	56.43
0s/1s	13.93	202.29
Neither	19.71	357.57

To follow up on this result, we examined students' time to correctly answer mixed facts according to their fluency level on each individual helper fact group. Table 4 shows average speed on mixed facts for students grouped according to their fluency level on 0s/1s facts. Students who are Fluent on 0s/1s are fastest on mixed facts, followed by students with Medium fluency, and then students with Slow fluency. An ANOVA detected a significant difference across groups ($p < 0.001$, F-value: 30.39, df: 2). Follow-up pairwise t-tests showed a significant difference between students with Slow fluency and each of the other two groups ($p < 0.001$), but no significant difference between Fluent students and students in the Medium fluency level. Therefore, some fluency on 0s/1s accompanies faster times to correctly answer mixed facts, but it does not seem to be advantageous to be fully Fluent rather than in the Medium fluency level.

Table 4. Speed on Mixed Facts by Fluency Level on 0s/1s.

Fluency Level on 0s/1s	**Mean Speed on Mixed Facts (s)**	**SD**
Fluent	10.54	135.76
Medium	14.26	125.57
Slow	26.93	599.43

Table 5 shows mean speed mixed facts for students grouped according to their fluency level on doubles. Students who are Fluent on doubles are fastest on mixed facts, followed by students with Medium fluency, and then students with Slow fluency. An ANOVA detected a significant difference across groups ($p < 0.001$, F-value: 51.33, df: 2). Follow-up pairwise t-tests showed a significant difference in all comparisons ($p < 0.001$). In contrast to 0s/1s, being Fluent on doubles appears to be more beneficial than being in the Medium fluency level, in terms of speed on mixed facts.

Table 5. Speed on Mixed Facts by Fluency Level on Doubles.

Fluency Level on Doubles	**Mean Speed on Mixed Facts (s)**	**SD**
Fluent	8.86	49.74
Medium	15.56	173.84
Slow	32.50	684.82

5. Educational or Scientific Importance of the Research

These findings have several implications for education. First, fluency with basic addition and subtraction facts in generally low. Students are generally not Fluent (fast and accurate) on any group of facts. Educators may need to spend more time on fact fluency development and/or adjust expectations for fact fluency, particularly as it pertains to speed in solving facts. Second, fluency with some facts is clearly acquired before others. The results of this study suggest that teachers may want to focus on the development of fluency with doubles facts and then with 0s/1s facts, as these appear to be acquired first. Finally, fluency with doubles, and to a lesser extent with 0s/1s, seems to aid fluency with other facts. Again, this suggests that teachers should encourage development of fluency with helper facts and then leverage that fluency to help students learn new facts.

6. Equity as a Collective Professional Responsibility

Research consistently indicates that struggles with fact fluency can cause lifelong underachievement in mathematics (Codding & Martin, 2016; Gersten et al., 2009). It is therefore critical to understand and better support students' early fact fluency development, so as to avoid later equity issues.

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